CONTROL IN DELAYED STOCHASTIC SYSTEMS

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Abstract

This article discusses issues of controlling in situations where fluctuations and delay are present. A new paradigm for control schemes, termed "delayed stochastic controls", is proposed for use in such situations. [1]

INTRODUCTION

Controlling under unpredictable and fast-changing situations, which has much shorter time constants than human reaction time, arises in a wide range of our daily activities. In some cases, such as car driving, it may constitute, at the same time, critical safety issues. Given the limitation of our feedback time and the existence of unpredictable fluctuations, we must employ rather complex controlling schemes combining our senses, feedbacks, and predictions. The main control schemes which have been investigated and developed so far are feedback and predictive controls, and they are successfully applied in a variety of engineering situations. However, are they sufficient in every situation including those described above? We conjecture that there may be other elements which come into control scheme in this kind of situation of controlling fast unpredictable moving objects with a system which has a longer feedback time.

Recently, an interesting experiment was performed which shed light on this issue[1, 2, 3]. The experiment is to measure the movement of a stick on a human fingertip in his effort of balancing it. The experiment has shown that the most of the fluctuating movement of the stick has a much shorter time constant compared to the typical human feedback time of few hundred milliseconds. Eventually, the stick falls off, but the experiment also shows that one can improve

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his balancing act by practice. It is unlikely that we can drastically improve our reaction time or predictive accuracy in this task. Our conjecture is that when one realizes that the stick motion is beyond his feedback time and his predictive accuracy, he may be bringing in another strategy. Probably unintentionally, one learns to adjust fingertip's fluctuation level given his feedback time limitation. In other words, one may be learning to add an appropriate level of noise that is adjusted according to one's delay time compared to the motion of the stick. We call this type of control scheme "Delayed Stochastic Control" [4]. It is the strategy aimed at obtaining an optimal combination of noise level and feedback delay time.

There is another experiment to show that an appropriate noise level externally given can help human posture control [5]. But, the relation of the noise level with the feedback delay time is not discussed there. We need to wait for more experimental evidence to support the existence of this new scheme of control.

In the rest of this paper, we discuss a simple mathematical toy model to give a preliminary support for this conjecture of delayed stochastic control. We also provide a brief report on an experiment to indicate an application of such control.

REPULSIVE DELAYED RANDOM WALK

As a mathematical framework to investigate the systems with noise and delay, delayed random walk has been proposed and studied[6, 7, 8]. This is a random walk whose transition probability depends on its position at a fixed time interval in the past. The focus has been placed on the model which has an attractive bias to a single point. This stable case has been applied to such processes like posture control [9]. Analytically, the attractive delayed random walk model has shown such behaviors like an oscillatory correlation function with increasing delay.

However, as the attractive model is not suitable to model the unstable situation we mentioned above, we discuss a delayed random walk which has a repulsive point. We can consider many different possibilities, but here we consider one-dimensional discrete time and step random walk with the origin as a repulsive point. Mathematically, we can define our model as follows. Let the position of the random walker at time step t given by $X(t)$ and the fixed point set at the origin, $X = 0$. The delayed random walk is defined by the following conditional probabilities.

$$P(X(t+1) = X(t)+1 | X(t-\tau) > 0) = p \qquad (1)$$

$$P(X(t+1) = X(t)+1 | X(t-\tau) = 0) = \frac{1}{2} \qquad (2)$$

$$P(X(t+1) = X(t)+1 | X(t-\tau) < 0) = 1-p, \qquad (3)$$

where $0 < p < 1$ and τ is the delay. With delay, the walker refers to its position in the past to decide on the bias of his next step. The attractive model is the case of $p < 0.5$, where the origin becomes attractive with no delay, $\tau = 0$. On the other hand $p > 0.5$ gives the repulsive case which we shall discuss for the rest of this paper.

Though this appears to be a little change of definition from the attractive case, we observe a very different behavior from the attractive case. Most of all, as the walker escapes away from the origin, we do not have a stationary probability distribution. This makes analytical treatment of this repulsive model more difficult as compared to the attractive case, particularly with non-zero delay. Our investigation in this paper is done by computer simulation. The most notable feature of this model is that we can find an optimal combination of the bias p and τ where the random walker can be kept around the origin for a longest duration.

ANALYSYS

As in the case of stick balance experiment, one of the main interests is how long the walker can be kept around the repulsive fixed point. We investigated this by focusing on an average first passage time L to reach a certain position (a limit point $\pm X^*$) away from the origin. In other words, we measured the average time for the walker starting from the origin to reach the limit point for the first time as we changed parameters in the model. The longer average first passage time indicates slower diffusion, which corresponds to the situation of longer stick balancing.

For the case of zero delay with the bias p, we can find an analytical result for this average first passage time L to reach the limit point $\pm X^*$ as

$$\langle L \rangle = 2\left(\frac{q}{q-p}\right)\left(\frac{1-\left(\frac{q}{p}\right)^{X^*}}{1-\frac{q}{p}}\right) + \frac{X^*}{p-q}, \quad (p \neq 0.5), \tag{4}$$

where we have set $q \equiv 1 - p$. For the case of simple (symmetric) random walk with $p = q = 0.5$, this result reduces to an even simpler form.

$$\langle L \rangle = (X^*)^2 \tag{5}$$

For the case of non-zero delay, such analytical result is yet to be obtained and computer simulation is used. We considered an ensemble of 10000 walkers. The initial condition is set so that the walker performs a normal random walk with no bias $p = 0.5$ for the duration of $t = (-\tau, 0)$. The walker's position at $t = 0$ is set as the origin $X = 0$. The limit point is set at $\pm X^*$. We measure the number of steps for each walker to go from the origin to $\pm X^*$ and average them. We performed computer simulations for various bias p and delay τ.

Some sample results are shown in Figure 1. The most notable features of these graphs are the peaks in the graph, indicating that the slowest diffusion appears at certain optimal values of τ given bias p. In other words, the walker is most stabilized around the origin with appropriate non-zero delay. This is rather unexpected result contrary to the normal notion associated with effects of feedback delay, where longer delay increasingly de-stabilize systems. Here, appropriate combination of bias and delay time is inducing more stability.

In order to gain more insight into this phenomenon, we look for an approximate analytical expression, which is found to be given by the following expressions

$$\langle L(\tau)\rangle = (1 + \alpha\tau_n e^{\beta\tau_n})\langle L(\tau = 0)\rangle. \tag{6}$$

Here α and β are parameters and τ_n is a normalized delay given as follows.

$$\tau_n \equiv \tau \frac{p-q}{X^*} \tag{7}$$

This normalization uses a characteristic time dividing the distance X^* by an average velocity of the walker $p-q$. Hence τ_n is a non-dimensionalized parameter as well.

Figure 2 shows this analytical approximation and the result of computer simulation. We see that the curves for various (p, X^*) overlaps quite well with the analytical curve with appropriately chosen parameters of $\alpha = 1.27$ and $\beta = 0.67$. We also notice that the peak height is approximately 1.7 times the average first passage time of zero delay case.

Stick Balancing Experiment with Motion

We performed the following experiment to gain some insight into the existence or utilization of delayed stochastic control. We asked the subjects to sit on a chair and balance a stick, as in the previous stick balancing experiment. But, this time, the subjects were allowed to move their bodies, not just their arms, as they tried to balance the stick. One way to do this is to hold an object with the other hand and move it (Figure 3). Another way is to move their legs. We measured the time for which they could keep the sticks balanced, and compared it with the normal non-movement situations. Out of the six subjects we tested, three subjects showed notable improvement in balancing by reaching their own optimal level of movement (Figure 4).

Some practice was needed for these subjects to reach this better performance. We believe that the subjects were tuning the appropriate level of fluctuation given their reaction times and prediction accuracy. Even though more thorough data needs to be collected, these results may be one supporting example of delayed stochastic control.

DISCUSSION

Relation of the model to Human Stick Balancing

The model we discussed in this paper is too simple to be a direct model of the stick balance experiment. Such properties like two scaling regions appeared in

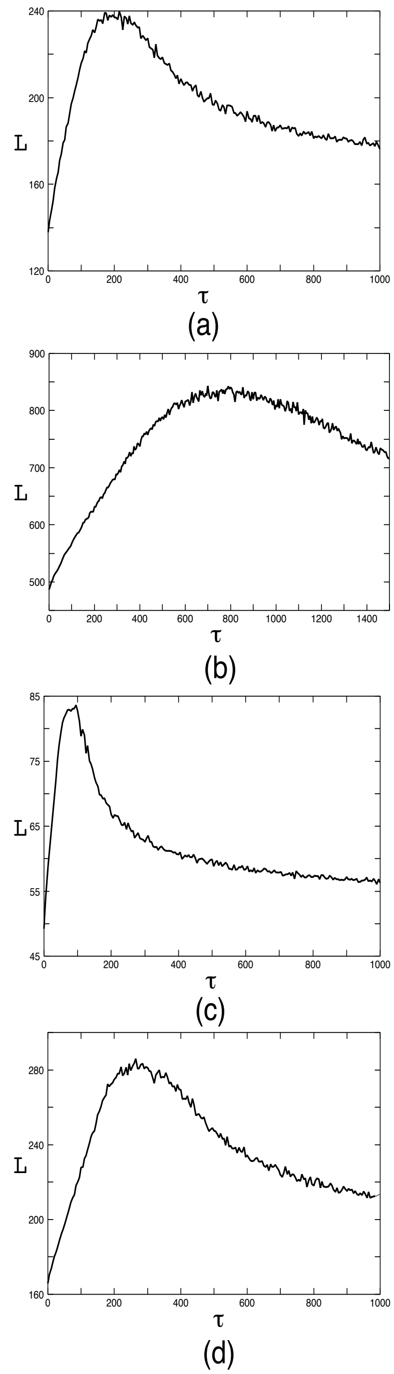

Figure 1: Average first passage time L as we change τ. The value of parameters (p, X^*) are (a) $(0.6, 30)$, (b) $(0.6, 100)$, (c) $(0.8, 30)$, and (d) $(0.8, 100)$.

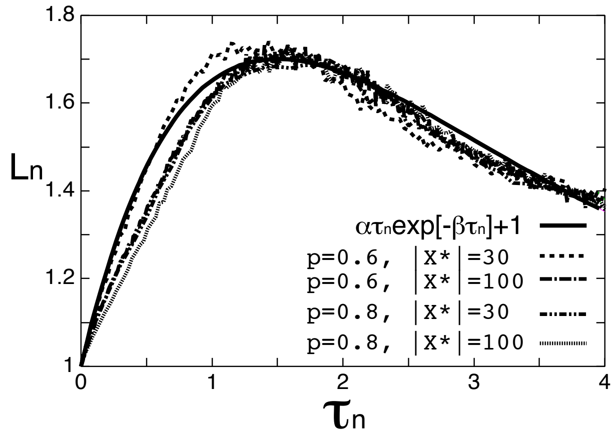

Figure 2: Normalized average first passage time L_n as we change normalized delay τ_n.The parameter sets (p, X^*) plotted are $(0.6, 30)$, $(0.6, 100)$, $(0.8, 30)$, and $(0.8, 100)$

the power spectra of experimental time series data can not be reproduced from time series of this repulsive delayed random walk. Yet, this model may shed some light to understanding the role of noise and delay in balancing.

It showed a behavior of "stabilization" at the optimal combination of bias p and delay τ. In the situation where noise level is constant, it may imply a completely new role of delay in the feedback systems: appropriate delay can induce more stabilization. Or, looking from another side, if the delay is constant, we can argue along the lines of "stochastic resonance" [13, 14, 15, 16, 17]. If we give appropriate bias, or fluctuations, we can obtain the best stabilization under constant delay. We have dealt with the notion of "resonance" with noise and delay with a simple two bit system[10, 11]. This is the first model, however, to indicate such notion may be useful for stabilization control.

From the experiment, it is reported that one can be better at stick balancing on the fingertip by practicing. It has been argued from the point of view of on-off intermittency that by practice one tunes parameters to the edge of dynamical instability. Our model implies an alternative view that by practice one tunes for optimal combination of noise level and delay for the best stabilization. Our preliminary experiment of incorporation of fluctuating motion during stick balancing indicates such possibility. It requires further investigation to understand the relations between these alternative view points.

Theoretical Issues

Analytical understanding of the model has a long way to go for the repulsive delayed random walk. We have looked at such statistical properties like laminar

Figure 3: Picture of a subject balancing a stick on one hand while moving an object in the other.

phases; probability distribution of the consequent time intervals for the walker to stay in $X < 0$ (or $X > 0$ by symmetry). It is possible to obtain generating functions for the case of zero delay, but not with non-zero delay. The same type of difficulty arises in understanding of "resonance" discussed here. In the case of simple two bit systems [10, 11] or attractive delayed random walks[12], some analytical insight has been obtained. We have found proper normalization to scale the phenomena here, but theoretical justification is still to be obtained.

Another important issue regarding delayed random walk is its effect of initial conditions. We have tried other conditions such as keeping all the walker's positions for $t = (-\tau, 0)$ to be positive, in which we again see an optimal stabilization. If we keep the walker at the origin for $t = (-\tau, 0)$, we see a monotone increase of first passage time with longer delay. Effect of the initial path is a theoretically difficult issue with the delay differential equations [18, 19]. The same difficulty appears in this model as well.

From a broader context, methodology and ideas developed in physical stochastic systems are mostly for the systems with attractive fixed points or fluctuation around stabilities. They are not always suited for dealing with unstable stochastic systems, such as the repulsive delayed random walk discussed here. Many of the time sequences observed in neural and physiological phenomena are not stationary and it is needless to say that such unstable stochastic systems are commonly seen in nature. It is hoped that this model will provide some contribution toward improving the understanding of such unstable stochastic systems.

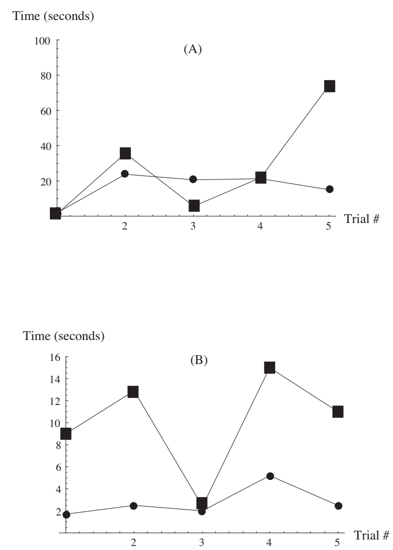

Figure 4: (A) Example of improvement on balancing tasks with (square) and without (dot) moving an object. The subject was given 5 trials without previous practice. By the 5th trial, the improvement was significant. (B) Another subject practiced for a few hours. Here, again improvement with moving the object was evident.

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References

[1] J. L. Cabrera and J. G. Milton: On-Off Intermittency in a Human Balancing Task, Physical Review Letters, **89** 158702 (2002).

[2] J. L. Cabrera and J. G. Milton: Tuning Levy Flights to Improve Balance Control, Chaos, **14** 691 (2004).

[3] J. L. Cabrera, R. Bormann, C. Eurich, T. Ohira and J. Milton: State–dependent Noise and Human Balance Control, Fluctuations Noise Letters, **4**, L107 (2004).

[4] T. Ohira and T. Hosaka: Repulsive Delayed Random Walk Artificial Life and Robotics, Vol.**9** (2005, in press).

[5] A. A. Priplata, J. B. Niemi, J. D. Harry, L. A. Lipsitz and J. J. Collins: Vibratory Insoles and Balance Control in Elderly People, Lancet, **362** 1123 (2003).

[6] T. Ohira and J. G. Milton: Delayed Random Walks, Physical Review E, **52** 3277 (1995).

[7] T. Ohira: Oscillatory Correlation of Delayed Random Walks, Physical Review E, **55** R1255 (1997).

[8] T. Ohira and T. Yamane: Delayed Stochastic Systems, Physical Review E, **61** 1247 (2000).

[9] J. J. Collins and C. J. DeLuca: Random Walking during Quiet Standing, Physical Review Letters, **73** 764 (1994).

[10] T. Ohira and Y. Sato: Resonance with Noise and Delay, Physical Review Letters, **82** 2811 (1999).

[11] T. Ohira, Y. Sato, and J. D. Cowan: Resonance in a Stochastic Neuron Model with Delayed Interaction, In *Advances in Neural Information Processing Systems 12* (S. A. Solla, T. K. Leen, and K-R. Muller eds.) pp. 314-320, MIT Press, 2000

[12] L.S. Tsimring and A. Pikovsky: Noise-induced dynamics in bistable systems with delay, Physical Review Letters, **87** 250602 (2001).

[13] A. Longtin, A. Bulsara, F. Moss: Phys. Rev. Lett. **67**, 656, (1991)

[14] K. Wiesenfeld, F. Moss: Nature, **373**, 33 (1995)

[15] J. J. Collins, C. C. Chow, T. T. Imhoff: Nature, **376**, 236 (1995)

[16] A. R. Bulsara, L. Gammaitoni: Physics Today, **49**, 3, 39 (1996)

[17] L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni: Rev. Mod. Phys. **70**, 223 (1998)

[18] M. C. Mackey, L. Glass: Science **197**, 287 (1977)

[19] K.L. Cooke, Z.Grossman: J. Math. Analysis and Applications **86**, 592 (1982)